Name: **Date:**

AP Calculus Summer Packet

***Please Note*: Your completed summer packet is due the *first day of school*, and you will be given a test on the material contained in the summer packet during a 75 minute class the *first week of school*. The summer packet will count for 10% and the test will count for 90% of your grade for the summer review.**

The following material was covered in Analysis H1. This material will not be covered in AP Calculus.. The assumption is that the material is part of your knowledge base. As a review, you are to complete the attached problems. If you have trouble completing any part of the assignment you are expected to seek help. Coolmath.com; Kahn Academy or YouTube can be used as a reference. Please do not wait until the last minute to complete the assignment.

Enclosed is a set of objectives on material that you should already know but that you might need to review. Along with the objectives is a reference guide with examples and explanations on each objective. Each objective has a set of problems for you to complete. You are expected to complete all the problems. An answer key to the odd numbered problems is attached. Be sure to check your odd numbered problems answers with the answer key.
Be sure to show all work for each problem.

Here is a list of links to help you with your assignments.

Functions: https://www.khanacademy.org/math/algebra2/manipulating-functions

Rational:https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions

Solving: https://www.khanacademy.org/math/algebra2/advanced-functions

Linear: https://www.khanacademy.org/math/algebra/two-var-linear-equations

Trig: https://www.khanacademy.org/math/precalculus/trig-equations-and-identities-precalc

- **Complex Fractions**

Example: Simplify $\dfrac{-7-\dfrac{6}{x+1}}{\dfrac{5}{x+1}}$

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}=\frac{\frac{-7(x+1)-6}{x+1}}{\frac{5}{x+1}}=\frac{\frac{-7x-7-6}{x+1}}{\frac{5}{x+1}}=\frac{\frac{-7x-13}{x+1}}{\frac{5}{x+1}}=\frac{-7x-13}{x+1}\cdot\frac{x+1}{5}=\frac{-7x-13}{5}$$

Exercises: Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$ **2.** $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$ **3.** $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$ **5.** $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

- **Compositions of Functions**

Example:

Given $f(x)=2x^2+1$ and $g(x)=x-4$, find $f(g(x))$ or $(f\circ g)(x)$.

$$f(g(x))=2(x-4)^2+1=2(x^2-8x+16)+1=2x^2-16x+32+1=2x^2-16x+33$$

Exercises: Let $f(x)=2x+1$ and $g(x)=2x^2-1$. Find each.

6. $f(2)$ **7.** $g(-3)$ **8.** $f(t+1)$

9. $f(g(-2))$ **10.** $g(f(m+2))$ **11.** $\dfrac{f(x+h)-f(x)}{h}$

Exercises: Let $f(x)=\sin x$. Find the exact solution.

12. $f\left(\dfrac{\pi}{2}\right)$ **13.** $f\left(\dfrac{2\pi}{3}\right)$

Exercises: Let $f(x)=x^2$, $g(x)=2x+5$, and $h(x)=x^2-1$. Find each.

14. $h(f(-2))$ **15.** $f(g(x-1))$ **16.** $g(h(x^2))$

Exercises: Find $\frac{f(x+h)-f(x)}{h}$ for the given function of f.

17. $f(x)=9x-3$ **18.** $f(x)=5-2x$

- **Intercepts**

Example:

Find the x and y intercept of $y=x^2-2x-3$

x-int: Let $y=0$, solve for x.

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=3$ and -1: (3,0) and (−1,0)

y-int: Let $x=0$, solve for y.

$y=(0)^2-2(0)-3$

$y=-3$

(0, −3)

Exercises: Find the x and y intercepts for each.

19. $y=2x-5$ **20.** $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$ **22.** $y^2=x^3-4x$

- **Systems**

Example: Find the point(s) of intersection of $x^2+y^2-16x+39=0$ and $x^2-y^2-9=0$

$$\begin{array}{r} x^2+y^2-16x+39=0 \\ +\ x^2-y^2-9=0 \\ \hline 2x^2-16x+30=0 \end{array}$$

$2(x^2-8x+15)=0 \rightarrow 2(x-5)(x-3)=0 \rightarrow x=5$ and $x=3$

$5^2-y^2-9=0 \rightarrow -y^2+16=0 \rightarrow 16=y^2 \rightarrow y=\pm 4$

$3^2-y^2-9=0 \rightarrow -y^2=0 \rightarrow y=0$

So, the points of intersection are: (5, 4), (5, −4), and (3, 0)

Exercises:

23. $\begin{array}{l} x+y=8 \\ 4x-y=7 \end{array}$

24. $\begin{array}{l} x^2+y=6 \\ x+y=4 \end{array}$

25. $\begin{array}{l} x^2-4y^2-20x-64y-172=0 \\ 16x^2+4y^2-320x+64y+1600=0 \end{array}$

- **Interval Notation**

Example:

Solution in set builder notation: $\{x|-5 \le x < 3\}$

Solution in interval notation: $[-5,3)$

Graph of solution:

Exercises: Fill in the table.

26.

Set Builder	Interval Notation	Graph
$\{x\|2 < x \le 4\}$		
	$[-1,7)$	
		

Exercises: Solve each equation. State your answer in both interval notation and graphically.

27. $2x-1 \ge 0$ **28.** $-4 \le 2x-3 < 4$ **29.** $\frac{x}{2}-\frac{x}{3} > 5$

- **Domain and Range**

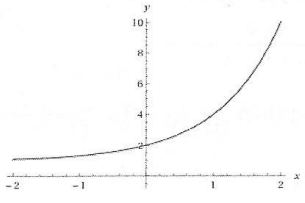
Example: Find the domain and range. Write your answer in interval notation.

$f(x)=3^x+1$

There are no domain restrictions.

There is a horizontal asymptote at $y = 1$.

Domain is $(-\infty, \infty)$: Range is $(1, \infty)$



Exercises: Find the domain and range of each. Write your answer in interval notation.

30. $f(x)=x^2-5$ **31.** $f(x)=-\sqrt{x+3}$ **32.** $f(x)=3\sin x$ **33.** $f(x)=\frac{2}{x-1}$

- **Inverses**

Example:

Find the inverse, $f(x)^{-1}$, of the given function $f(x)=\sqrt[3]{x+1}$. Switch x and y and solve for y.

$$f(x)=\sqrt[3]{x+1} \rightarrow y=\sqrt[3]{x+1} \rightarrow x=\sqrt[3]{y+1} \rightarrow x^3=y+1 \rightarrow x^3-1=y \rightarrow f(x)^{-1}=x^3-1$$

Exercises: Find the inverse of each.

34. $f(x)=2x+1$ **35.** $f(x)=\frac{x^2}{3}$

Example:

If two functions are inverses of each other, then $f(g(x))=x$ or $g(f(x))=x$.

Prove $f(x)=\frac{x-9}{4}$ and $g(x)=4x+9$ are inverses of each other.

$$f(g(x))=\frac{(4x+9)-9}{4}=\frac{4x}{4}=x \ \text{☺}$$

$$g(f(x))=4\left(\frac{x-9}{4}\right)+9=x-9+9=x \ \text{☺}$$

Exercises: Prove f and g are inverses of each other through compositions.

36. $f(x)=\frac{x^3}{2}, g(x)=\sqrt[3]{2x}$ **37.** $f(x)=9-x^2, g(x)=\sqrt{9-x}$ for $x \le 9$.

- **Equation of a Line**

Example:

Find the equation of a line that has slope $m = 3$ and goes through $(4, -2)$.

$$y - y_1 = m(x - x_1) \rightarrow y + 2 = 3(x - 4) \rightarrow y + 2 = 3x - 12 \rightarrow y = 3x - 14$$

Exercises: Find the equation of the line, in slope-intercept form ($y = mx + b$), being described.

38. Has a slope of 3 and a y-intercept of 5.

39. Passing through $(5, -3)$ with undefined slope.

40. Goes through $(-4, 2)$ with a slope of 0.

41. Goes through $(0, 5)$ with a slope of $\frac{2}{3}$.

42. Goes through $(2, 8)$ and is parallel to $y = \frac{5}{6}x - 1$.

43. Perpendicular to the y axis and goes through $(4, 7)$.

44. Goes through $(-3, 6)$ and $(1,2)$.

45. Has an x-intercept of $(2, 0)$ and a y-intercept of $(0, 3)$.

- **Graphing Calculator**

Example:

In AP Calculus, students will need to comfortable using a graphing calculator. Things that will need to be known include, but are not limited to: Graphing functions, finding roots/zeros, finding intersections between two functions, finding numerical derivatives, and finding numerical integrals.

2^{nd} + Trace → option 2: zero → left bound: enter → right bound: enter → guess: enter

Exercises: Find all zeros of each function on the given interval.

46. $f(x)=x^4-3x^3+2x^2-7x-11$ on $(-\infty, \infty)$

47. $f(x)=3\sin 2x-4x+1$ on $[-2\pi, 2\pi]$

48. $f(x)=0.7x^2+3.2x+1.5$ on $(-\infty, \infty)$

49. $f(x)=x^4-8x^2+5$ on $(-\infty, \infty)$

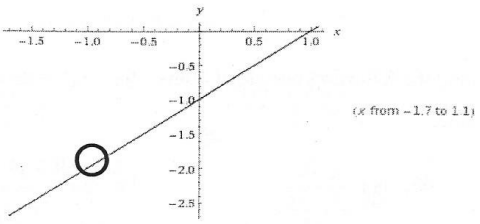
Exercises: Find all points of intersection.

50. $f(x)=x^2-5x+2$ and $g(x)=3-2x$

- **Limits**

Examples: Limit at a point.

Find $\lim\limits_{x\to -1}\dfrac{x^2-1}{x+1}=\dfrac{(x+1)(x-1)}{x+1}=-2$



Exercises: Complete the table to estimate the limit.

51. $\lim\limits_{x\to 4}\dfrac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

52. $\lim\limits_{x\to -5}\dfrac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
$f(x)$						

Exercises: Find the limits graphically.

53. $\lim_{x\to 0} \cos x$

54. $\lim_{x\to 5} \dfrac{2}{x-5}$

55. $\lim_{x\to 1} \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$

Exercises: Find the limit analytically.

56. $\lim_{x\to 2} \left(4x^2+3\right)$

57. $\lim_{x\to 1} \left(\dfrac{x^2+x+2}{x+1}\right)$

58. $\lim_{x\to 0} \sqrt{x^2+4}$

59. $\lim_{x\to \pi} \cos x$

60. $\lim_{x\to 1} \left(\dfrac{x^2-1}{x-1}\right)$

61. $\lim_{x\to -3} \left(\dfrac{x^2+x-6}{x+3}\right)$

62. $\lim_{x\to 0} \left(\dfrac{\sqrt{x+1}-1}{x}\right)$

63. $\lim_{h\to 0} \left(\dfrac{2(x+h)-2x}{h}\right)$

Exercises: Find the following one-sided limits. $\lim_{x\to c^+} f(x)$ = from the right side. $\lim_{x\to c^-} f(x)$ = from the left side.

65. $\lim_{x\to 5^+} \dfrac{x-5}{x^2-25}$

66. $\lim_{x\to 3^-} \dfrac{x}{\sqrt{x^2-9}}$

67. $\lim_{x\to 10^+} \dfrac{|x-10|}{x-10}$

68. $\lim_{x\to 5^-} \left(\dfrac{-3}{x+5}\right)$

- **Asymptotes**

Example:

Find all asymptotes for the following: $f(x)=\dfrac{6x^2-x-2}{2x^2+9x+4}$.

Vertical asymptotes: set denominator = 0 and solve. $2x^2+9x+4=0 \rightarrow (2x+1)(x+4)=0$
There are two possible vertical asymptotes. $x=-½$ and $x=-4$. I say possible because if the numerator factors, it may cancel a factor in the denominator and turn the asymptote into a hole.

$f(x)=\dfrac{6x^2-x-2}{2x^2+9x+4}=\dfrac{(2x+1)(3x-2)}{(2x+1)(x+4)}=\dfrac{3x-2}{x+4}$ → the $(2x+1)$'s canceled, $x=-½$ is a hole. $x=-4$ is the only vertical asymptote.

Horizontal asymptotes: take $\lim\limits_{x\to\infty} f(x)$. $\lim\limits_{x\to\infty}\dfrac{6x^2-x-2}{2x^2+9x+4}=\dfrac{\frac{6x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}{\frac{2x^2}{x^2}+\frac{9x}{x^2}+\frac{4}{x^2}}=\dfrac{6+0-0}{2+0-0}=3$. There is a horizontal asymptote at $y=3$.

Case 1 = degree in numerator is higher than degree in denominator. NO ASYMPTOTES
Case 2 = degree in numerator is lower than degree in denominator. $y=0$.
Case 3 = degree in numerator is same as degree in denominator. y = ratio of lead coefficients.

Exercises: Find all vertical asymptotes and/or holes for the following.

69. $f(x)=\dfrac{1}{x^2}$ **70.** $f(x)=\dfrac{x^2}{x^2-4}$ **71.** $f(x)=\dfrac{2+x}{x^2(1-x)}$

Exercises: Find all horizontal asymptotes for the following.

72. $f(x)=\dfrac{x^2-2x+1}{x^3+x-7}$ **73.** $f(x)=\dfrac{5x^3-2x^2+8}{4x-3x^3+5}$ **74.** $f(x)=\dfrac{4x^5}{x^2-7}$

Exercises: Find the limit as x approaches infinity.

75. $\lim\limits_{x\to\infty}\left(\dfrac{2x-5+4x^2}{3-5x+x^2}\right)$ **76.** $\lim\limits_{x\to\infty}\left(\dfrac{2x-5}{3-5x+3x^2}\right)$ **77.** $\lim\limits_{x\to\infty}\left(\dfrac{7x+6-2x^3}{3+14x+x^2}\right)$

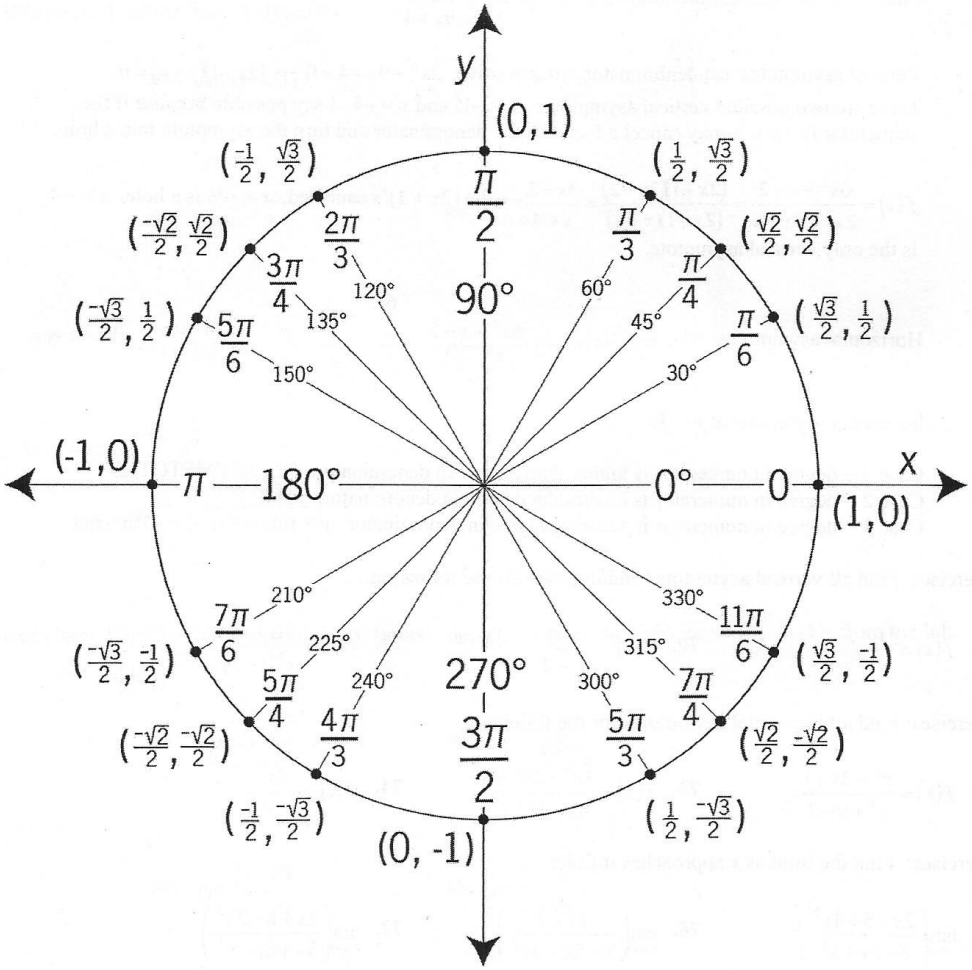
- **Infinite Limits**

Exercises: Determine whether these limits approach $\pm\infty$.

78. $\lim\limits_{x\to-1^+}\left(\dfrac{1}{x+1}\right)$ **79.** $\lim\limits_{x\to1^+}\left(\dfrac{2+x}{1-x}\right)$ **80.** $\lim\limits_{x\to0}\left(\dfrac{2}{\sin x}\right)$

- **Unit Circle**

All AP students need to have a working knowledge of the unit circle for its many applications to trigonometry.



Exercises: Using the unit circle find all the exact values of each.

81. $\sin(0)=$

82. $\sin(\pi/6)=$

83. $\cos(0)=$

84. $\cos(5\pi/6)=$

85. $\sin(3\pi/2)=$

86. $\tan(3\pi/2)=$

87. $\cos(\pi)=$

88. $\sin(\pi/4)=$

89. $\sin(5\pi/2)=$

90. $\cos(11\pi/6)=$

91. $\tan(0)=$

92. $\sin(\pi)=$

- **Simplifying Expressions**

Exercises: Simplify each expression.

93. $\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{}$

94. $\dfrac{\dfrac{2}{x^2}}{\dfrac{10}{x^3}}$

95. $\dfrac{12x^{-3}y^2}{18xy^{-1}}$

96. $\dfrac{15x^2}{5\sqrt{x}}$

97. $(5a^3)(4a^2)$

98. $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

99. $\dfrac{\dfrac{1}{2}-\dfrac{5}{4}}{\dfrac{3}{8}}$

100. $\dfrac{5-x}{x^2-25}$

ODD ANSWERS:

1. $\frac{5-a}{a}$

3. $\frac{4x-12}{5x}$

5. $\frac{x-4}{3x^2-4x+32}$

7. 17

9. 15

11. 2

13. $\frac{\sqrt{3}}{2}$

15. $(2x+3)^2$

17. 9

19. $(5/2,0)$ and $(0,-5)$

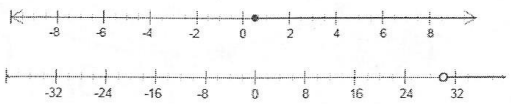
21. $(0,0)$ and $(-4,0)$ and $(4,0)$

23. $(3,5)$

25. $(6,-8)$ and $(14,-8)$

27. $[1/2,\infty)$

29. $(30,\infty)$



31. Domain = $[-3,\infty)$ Range = $(-\infty,0]$

33. Domain = $(-\infty,1)$ and $(1,\infty)$ Range = $(-\infty,0)$ and $(0,\infty)$

35. $\pm\sqrt{3x}$

37. $f(g(x))=g(f(x))=x$

39. $x=5$

41. $y=2$

43. $y=7$

45. $y=(-3/2)x+3$

47. $x=0.957$

49. $x=-2.705, -0.827, 0.827, 2.705$

51. $f(x)=0.20408,0.2004,0.20004,0.19996,0.1996,0.19608$
Limit = 0.2

53. 1

55. 4

57. 2

59. -1

61. -5

63. $-1/6$

65. 1/10

67. 1

69. $x=0$

71. $x=0, 1$

73. $y=-5/3$

75. 4

77. $-\infty$

79. DNE

81. 0

83. 1

85. -1

87. -1

89. 1

91. 0

93. $\dfrac{-h}{x^2+xh}$

95. $\dfrac{2y^3}{3x^4}$

97. $20a^5$

99. -2